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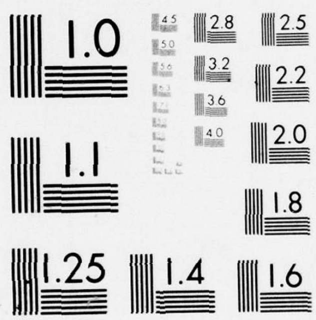

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ON THE SPEED OF PROPAGATION OF WAVES IN A DEFORMED COMPRESSIBLE ELASTIC MATERIAL

by

K.N. SAWYERS and R.S. RIVLIN

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On the Speed of Propagation of Waves in a Deformed Compressible Elastic Material

by

K.N. Sawyers and R.S. Rivlin
Center for the Application of Mathematics
Lehigh University, Bethlehem, Pa.

Abstract

A sinusoidal wave of infinitesimal amplitude is propagated in an isotropic compressible elastic material subjected to a static pure homogeneous deformation. Necessary and sufficient conditions are obtained for the speed of propagation to be real in the case when the direction of propagation is parallel to a principal plane of the static deformation.

1. Introduction

This paper is concerned with the conditions which must be imposed on the strain-energy function for an isotropic compressible elastic material in order to ensure that, when the material is subjected to a static pure homogeneous deformation, plane sinusoidal waves of infinitesimal amplitude propagate in it with real velocities. The problem has previously been considered by Hayes and Rivlin [1]. They obtained, in implicit form, necessary and sufficient conditions for the velocities to be real for arbitrary direction of propagation of the wave. These are the inequalities given in (2.10) below. Certain necessary conditions were also obtained in [1,2]. These are the inequalities given in (3.6) and (3.17) below.

Earlier Ericksen [3] considered the propagation of a second-order discontinuity in an isotropic incompressible material, rather than the substantially equivalent problem of the propagation of a sinusoidal wave of infinitesimal amplitude. He obtained, in explicit form, the necessary and sufficient conditions that waves propagated in the principal directions have real velocities.

Sawyers and Rivlin [4] have obtained, again in explicit form, necessary and sufficient conditions that the velocities of sinusoidal waves of infinitesimal amplitude be real for all directions of propagation parallel to a principal plane of the pure homogeneous deformation for an incompressible material. More recently they discussed [5] the implications of these results with respect to propagation in an arbitrary direction, not necessarily parallel to a principal plane.

In the present paper, the results of Hayes and Rivlin [1] for compressible materials are used to derive, in explicit form, necessary and sufficient conditions that the wave velocities be real for any direction of propagation parallel to a principal plane of an underlying static pure homogeneous deformation. These are the relations given by (3.6), (3.15), and (3.17) below.

Recently, Knowles and Sternberg [6] have obtained the necessary and sufficient conditions that the incremental equations of equilibrium for planar infinitesimal static deformations superposed on a plane pure homogeneous deformation be strongly elliptic. Since the conditions for reality of wave speeds are equivalent to the conditions that the incremental equations of equilibrium be strongly elliptic, the conditions obtained by Knowles and Sternberg should be equivalent to those which we have obtained, if, in our results, we take the principal extension ratio in a direction normal to the plane of propagation to be unity. That this is indeed the case is shown in §4.

2. Basic equations

We consider an isotropic elastic material to be subjected to a deformation in which a particle initially in vector position $\underset{\sim}{\xi}$ moves to vector position $\underset{\sim}{x}$. Let ξ_α and x_i be the components of $\underset{\sim}{\xi}$ and $\underset{\sim}{x}$ respectively in a rectangular cartesian coordinate system x. The strain-energy function w, per unit volume of material measured in the undeformed state, is expressible as a function of the three strain invariants i_1, i_2, i_3 defined by

$$i_1 = \mathrm{tr}\, \underset{\sim}{C}\ , \quad i_2 = \tfrac{1}{2}\{(\mathrm{tr}\, \underset{\sim}{C})^2 - \mathrm{tr}\, \underset{\sim}{C}^2\}\ , \quad i_3 = \det \underset{\sim}{C}, \tag{2.1}$$

where $\underset{\sim}{C}$ is the Cauchy strain, defined by*

$$\underset{\sim}{C} = \| C_{\alpha\beta} \|\ , \quad C_{\alpha\beta} = x_{i,\alpha} x_{i,\beta}\ , \tag{2.2}$$

We now consider that the deformation $\underset{\sim}{\xi} \rightarrow \underset{\sim}{x}$ is the resultant of a pure homogeneous deformation $\underset{\sim}{\xi} \rightarrow \underset{\sim}{X}$ and an infinitesimal deformation $\underset{\sim}{X} \rightarrow \underset{\sim}{x}$ where

$$\underset{\sim}{x} = \underset{\sim}{X} + \varepsilon \underset{\sim}{u}\ . \tag{2.3}$$

We suppose that the principal extension ratios for the pure homogeneous deformation are λ_1, λ_2, λ_3 and the principal directions are parallel to the axes of the system x.

We adopt the notation

$$w_A = \partial w/\partial i_A\ , \quad w_{AB} = \partial^2 w/\partial i_A \partial i_B\ ,$$
$$I_A = i_A\big|_{\varepsilon=0}\ , \quad W_A = w_A\big|_{\varepsilon=0}\ , \quad W_{AB} = w_{AB}\big|_{\varepsilon=0}$$
$$(A,B = 1,2,3)\ . \tag{2.4}$$

---

* In this paper we use the Einstein summation convention for repeated lower case greek or latin subscripts, but not for upper case latin subscripts.

Then,

$$I_1 = \lambda_1^2+\lambda_2^2+\lambda_3^2 \ , \quad I_2 = \lambda_2^2\lambda_3^2 + \lambda_3^2\lambda_1^2 + \lambda_1^2\lambda_2^2 \ ,$$

$$I_3 = \lambda_1^2\lambda_2^2\lambda_3^2 \ . \tag{2.5}$$

We now suppose that the deformation $\underset{\sim}{X} \to \underset{\sim}{x}$ corresponds to a plane sinusoidal wave of infinitesimal amplitude propagating in the material in the direction of the unit vector $\underset{\sim}{\ell}$. Then, with the usual complex notation we may write

$$\underset{\sim}{u} = \underset{\sim}{U} \exp \iota\omega(S\underset{\sim}{\ell}\cdot\underset{\sim}{X}-t) \ , \tag{2.6}$$

where ω is the angular frequency of the wave, S is its complex slowness and $\underset{\sim}{U}$ is a constant vector, which may be complex.

It has been shown by Hayes and Rivlin [1] that $\underset{\sim}{U}$ must satisfy the equation

$$\sum_{A=1}^{3} (s_{AB}-C^2\delta_{AB})U_B = 0 \ , \tag{2.7}$$

where

$$C^2 = \frac{1}{2} I_3^{\frac{1}{2}} \rho/S^2 \ ,$$

$$s_{AB} = (\lambda_A^2\lambda_B^2W_2 + I_3W_3)\ell_A\ell_B$$

$$+ \delta_{AB}\{\sum_{C=1}^{3} \lambda_C^2\ell_C^2[W_1+W_2(I_1-\lambda_A^2-\lambda_C^2)]\}$$

$$+ c_{AB}\ell_A\ell_B$$

and (2.8)

$$c_{AB} = 2\{\lambda_A^2[W_{11}+(I_1-\lambda_A^2)W_{21}] + I_3W_{31}\}\lambda_B^2$$

$$+ 2\{\lambda_A^2[W_{12}+(I_1-\lambda_A^2)W_{22}] + I_3W_{32}\}\lambda_B^2(I_1-\lambda_B^2)$$

$$+ 2\{\lambda_A^2[W_{13}+(I_1-\lambda_A^2)W_{23}] + I_3W_{33}\}I_3 \ .$$

ρ denotes the density of the material in the undeformed state.

The condition for (2.7) to yield a non-trivial solution for $\underset{\sim}{U}$ is

$$\left| s_{AB} - C^2 \delta_{AB} \right| = 0 . \tag{2.9}$$

This is the secular equation for the wave.

We note that since the matrix $\underset{\sim}{s} = \| s_{AB} \|$ is real and symmetric, the values of C^2 given by (2.9) are necessarily real.

The necessary and sufficient conditions that equation (2.9) yield three positive solutions for C^2 are [1]

$$\mathrm{tr}\, \underset{\sim}{s} > 0 , \quad \{(\mathrm{tr}\, \underset{\sim}{s})^2 - \mathrm{tr}\, \underset{\sim}{s}^2\} > 0 , \quad \det \underset{\sim}{s} > 0 . \tag{2.10}$$

From (2.8) we have

$$\mathrm{tr}\, \underset{\sim}{s} = \sum_{A=1}^{3} s_{AA} = \sum_{A=1}^{3} \{\lambda_A^2[3W_1 + 2(I_1 - \lambda_A^2)W_2] + I_3 W_3 + c_{AA}\} \ell_A^2 , \tag{2.11}$$

where

$$\begin{aligned} c_{AA} = 2\{&\lambda_A^4[W_{11} + 2(I_1 - \lambda_A^2)W_{12} + (I_1 - \lambda_A^2)^2 W_{22}] \\ &+ I_3[2\lambda_A^2 W_{31} + 2\lambda_A^2(I_1 - \lambda_A^2)W_{32} + I_3 W_{33}]\} . \end{aligned} \tag{2.12}$$

With (2.11) it is seen that the necessary and sufficient conditions that $(2.10)_1$ be satisfied for all $\underset{\sim}{\ell}$ are

$$\lambda_A^2[3W_1 + 2(I_1 - \lambda_A^2)W_2] + I_3 W_3 + c_{AA} > 0 \qquad (A = 1,2,3) . \tag{2.13}$$

3. Propagation in a principal plane

We now suppose that the direction of propagation of a wave lies in a principal plane, say parallel to the x_1x_2-plane of the coordinate system. Then, $\underset{\sim}{\ell} = (\ell_1,\ell_2,0)$ and from (2.8) we obtain

$$\begin{aligned}
s_{11} &= \lambda_1^2\ell_1^2\{K_3+\lambda_2^2L_3 + (c_{11}/\lambda_1^2)\} + \lambda_2^2\ell_2^2K_3 , \\
s_{22} &= \lambda_2^2\ell_2^2\{K_3+\lambda_1^2L_3 + (c_{22}/\lambda_2^2)\} + \lambda_1^2\ell_1^2K_3 , \\
s_{33} &= \lambda_1^2\ell_1^2K_2 + \lambda_2^2\ell_2^2K_1 , \\
s_{12} &= s_{21} = \lambda_1^2\lambda_2^2\ell_1\ell_2\{L_3+(c_{12}/\lambda_1^2\lambda_2^2)\} , \\
s_{23} &= s_{32} = s_{31} = s_{13} = 0 ,
\end{aligned} \tag{3.1}$$

with the notation

$$K_A = W_1+ \lambda_A^2W_2 , \quad L_A = W_2+ \lambda_A^2W_3 . \tag{3.2}$$

With (3.1) the secular equation (2.9) yields

$$C^2 = s_{33} , \tag{3.3}$$

or

$$(C^2-s_{11})(C^2-s_{22}) - s_{12}^2 = 0 . \tag{3.4}$$

The necessary and sufficient conditions that (3.3) yield only positive values for C^2 for all ℓ_1,ℓ_2 are the two Baker-Ericksen conditions

$$K_1 > 0 , \quad K_2 > 0 . \tag{3.5}$$

Analogous considerations for waves propagated in the x_2x_3 or

x_3x_1 planes lead additionally to the remaining Baker-Ericksen condition $K_3 > 0$. We accordingly have

$$K_A > 0 \quad (A = 1,2,3) . \tag{3.6}$$

The necessary and sufficient conditions that (3.4) yield only positive values for C^2 are

$$s_{11} + s_{22} > 0 \quad \text{and} \quad s_{11}s_{22} - s_{12}^2 > 0 . \tag{3.7}$$

From (3.1), we obtain

$$\begin{aligned} s_{11} + s_{22} &= \lambda_1^2 \ell_1^2 (F_1 + K_3) + \lambda_2^2 \ell_2^2 (F_2 + K_3) , \\ s_{11}s_{22} - s_{12}^2 &= K_3 (\lambda_1^4 \ell_1^4 F_1 + \lambda_2^4 \ell_2^4 F_2) + 2\lambda_1^2 \lambda_2^2 \ell_1^2 \ell_2^2 G_3 , \end{aligned} \tag{3.8}$$

where

$$\begin{aligned} F_1 &= K_3 + \lambda_2^2 L_3 + c_{11}/\lambda_1^2 , \quad F_2 = K_3 + \lambda_1^2 L_3 + c_{22}/\lambda_2^2 , \\ 2G_3 &= F_1 F_2 + K_3^2 - \lambda_1^2 \lambda_2^2 \left(L_3 + \frac{c_{12}}{\lambda_1^2 \lambda_2^2} \right)^2 , \end{aligned} \tag{3.9}$$

With $(3.8)_1$, we see that the necessary and sufficient conditions for $(3.7)_1$ to be satisfied for all ℓ_1, ℓ_2 are

$$F_1 + K_3 > 0 \quad \text{and} \quad F_2 + K_3 > 0 . \tag{3.10}$$

With (3.9) and (3.2), these conditions may be rewritten as the first two of the conditions

$$\lambda_A^2 [2W_1 + (2\lambda_C^2 + \lambda_B^2) W_2] + I_3 W_3 + c_{AA} > 0$$
$$(ABC = 123,\ 213,\ 231,\ 321,\ 312,\ 132) . \tag{3.11}$$

The remaining four conditions may be obtained, in a manner analogous to that employed in obtaining the first two, by considering waves to be propagated parallel to the x_2x_3 and x_3x_1 planes. The conditions (3.11) imply the conditions (2.13) provided that the conditions (3.6) are satisfied.

From $(3.8)_2$, it follows that necessary conditions that the inequality $(3.7)_2$ be satisfied for all ℓ_1, ℓ_2 are*

$$F_1 > 0 \ , \quad F_2 > 0 \ . \tag{3.12}$$

With (3.6) it follows that the conditions (3.12) imply (3.10).

With these conditions we may rewrite $(3.8)_2$ as

$$\begin{aligned} s_{11}s_{22} - s_{12}^2 = K_3\{ & (\lambda_1^2\ell_1^2F_1^{\frac{1}{2}} - \lambda_2^2\ell_2^2F_2^{\frac{1}{2}})^2 \\ & + 2\lambda_1^2\lambda_2^2\ell_1^2\ell_2^2(F_1^{\frac{1}{2}}F_2^{\frac{1}{2}} + G_3/K_3)\} \ . \end{aligned} \tag{3.13}$$

It follows that, provided the conditions (3.6) and (3.12) are satisfied, the necessary and sufficient condition for $(3.7)_2$ to be satisfied is

$$K_3F_1^{\frac{1}{2}}F_2^{\frac{1}{2}} + G_3 > 0 \ . \tag{3.14}$$

With (3.9), the condition (3.14) can be rewritten as the first of the conditions

$$F_A^{\frac{1}{2}}F_B^{\frac{1}{2}} + K_C > \lambda_A\lambda_B\left|L_C + \frac{c_{AB}}{\lambda_A^2\lambda_B^2}\right| \qquad (ABC = 123,\ 231,\ 312) \ , \tag{3.15}$$

---

* The conditions (3.12) have been previously obtained [1,2] as necessary conditions that waves propagated in an arbitrary direction, not necessarily parallel to a principal plane, have real speeds.

where, from (3.2) and (2.8),

$$
\begin{aligned}
K_C &= W_1 + \lambda_C^2 W_2 , \quad L_C = W_2 + \lambda_C^2 W_3 , \\
F_A &= W_1 + (I_1 - \lambda_A^2) W_2 + \frac{I_3}{\lambda_A^2} W_3 + \frac{c_{AA}}{\lambda_A^2} , \\
\frac{1}{2} c_{AB} &= \lambda_A^2 \lambda_B^2 \{ W_{11} + (2I_1 - \lambda_A^2 - \lambda_B^2) W_{12} + (I_1 - \lambda_A^2)(I_1 - \lambda_B^2) W_{22} \} \\
&\quad + I_3 \{ (\lambda_A^2 + \lambda_B^2) W_{31} + [\lambda_A^2 (I_1 - \lambda_A^2) + \lambda_B^2 (I_1 - \lambda_B^2)] W_{32} + I_3 W_{33} \} .
\end{aligned}
\tag{3.16}
$$

Also, the conditions (3.12) are the first two of the conditions

$$F_A > 0 \quad (A = 1,2,3) . \tag{3.17}$$

The second and third of the conditions (3.16) and the third of the conditions (3.17) may be obtained in an analogous manner by considering waves to be propagated parallel to the $x_2 x_3$ and $x_3 x_1$ planes respectively.

We conclude that the necessary and sufficient conditions that waves propagated parallel to a principal plane of the pure homogeneous deformation shall have real velocities are those in equations (3.15), (3.6) and (3.17).

4. A particular case: plane strain

In this section, we will consider the particular case when $\lambda = 1$. It will be shown that this yields conditions on the strain-energy function equivalent to those obtained by other considerations by Knowles and Sternberg [6].

We write

$$i_1 = i + 1, \quad i_3 = j^2, \quad i_2 = i + j^2 . \tag{4.1}$$

Then, writing $I = i\big|_{\varepsilon=0}$, $J = j\big|_{\varepsilon=0}$, we have

$$I = I_1 - 1 = \lambda_1^2 + \lambda_2^2 \quad , \; J = I_3^{1/2} = \lambda_1\lambda_2 \; , \; I_2 = I + J^2 . \tag{4.2}$$

We write $\phi(i,j) = w(i_1,i_2,i_3)$ and introduce the notation (cf.(2.4))

$$\begin{aligned} \Phi_I &= \partial\phi/\partial i\big|_{\varepsilon=0} \qquad \Phi_J = \partial\phi/\partial j\big|_{\varepsilon=0} , \\ \Phi_{II} &= \partial^2\phi/\partial i^2\big|_{\varepsilon=0} , \quad \text{etc.} \end{aligned} \tag{4.3}$$

With $\lambda_3 = 1$, we obtain

$$\begin{aligned} \Phi_I &= W_1 + W_2 \; , \quad \Phi_J = 2J(W_2 + W_3) \; , \\ \Phi_{II} &= W_{11} + 2W_{12} + W_{22} \; , \\ \Phi_{IJ} &= 2J(W_{12} + W_{13} + W_{22} + W_{23}) \; , \\ \Phi_{JJ} &= 2(W_2 + W_3) + 4J^2(W_{22} + 2W_{23} + W_{33}) \; . \end{aligned} \tag{4.4}$$

Taking $\lambda_3 = 1$ in (3.16) and using the relations (4.4) we obtain

$$
\begin{aligned}
K_3 &= \Phi_I \ , \quad L_3 = \frac{1}{2J}\Phi_J \ , \\
F_1 &= \Phi_I + 2\lambda_1^2\Phi_{II} + 2J\Phi_{IJ} + \frac{1}{2}\lambda_2^2\Phi_{JJ} \ , \\
F_2 &= \Phi_I + 2\lambda_2^2\Phi_{II} + 2J\Phi_{IJ} + \frac{1}{2}\lambda_1^2\Phi_{JJ} \ , \\
\frac{C_{12}}{\lambda_1^2\lambda_2^2} &= 2\Phi_{II} + \frac{I}{J}\Phi_{IJ} + \frac{1}{2}\left(\Phi_{JJ} - \frac{\Phi_J}{J}\right) .
\end{aligned}
\tag{4.5}
$$

With (4.4) and (4.5), the relations (3.6), (3.12) and (3.15) yield the strong ellipticity conditions of Knowles and Sternberg [6]

$$
\begin{aligned}
&B > 0 \ , \quad E_{11} > 0 \ , \quad E_{22} > 0 \ , \\
&E_{11}^{1/2}E_{22}^{1/2} + E_{12} > 0 \ ,
\end{aligned}
\tag{4.6}
$$

where

$$
\begin{aligned}
B &= 2\Phi_I \ , \ E_{11} = 4\Phi_I F_1 \ , \ E_{22} = 4\Phi_I F_2 \ , \\
E_{12} &= 2\Phi_I(F_1+F_2) + 2(\lambda_1^2-\lambda_2^2)^2(\Phi_{II}\Phi_{JJ} - \Phi_{IJ}^2) .
\end{aligned}
\tag{4.7}
$$

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References

[1] M. Hayes and R.S. Rivlin, Arch. Rational Mech. Anal., **8**, 15 (1961).

[2] R.S. Rivlin, Foundations of Continuum Thermodynamics, ed. N. Nina and J. Whitelaw; McMillan, London, 229 (1975).

[3] J.L. Ericksen, J. Rational Mech. Anal., **2**, 329 (1953).

[4] K.N. Sawyers and R.S. Rivlin, Int. J. Solids Structures, **9**, 607 (1973).

[5] K.N. Sawyers and R.S. Rivlin, Z. Angew. Math. u. Phys. (in the press)

[6] J.K. Knowles and E. Sternberg, Arch. Rational Mech. Anal., **63**, 321 (1977).

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Polytechnic Institute of New York
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333 Jay Street
Brooklyn, NY 11201

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Polytechnic Institute of New York
Department of Aerospace Engineering and
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333 Jay Street
Brooklyn, NY 11201

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Los Angeles, CA 90024

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Chemistry Road
Buffalo, NY 14214

Professor Joseph L. Rose
Drexel University
Department of Mechanical Engineering and Mechanics
Philadelphia, PA 19104

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University of California, San Diego
Department of Chemistry
La Jolla, CA 92093

Professor B. K. Donaldson
University of Maryland
Aerospace Engineering Department
College Park, MD 20742

Professor Joseph A. Clark
Catholic University of America
Department of Mechanical Engineering
Washington, DC 20064

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Cambridge, MA 02138

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General Dynamics Corporation
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Groton, CT 06340

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J. G. Engineering Research Associates
3831 Menlo Drive
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Southwest Research Institute
8500 Culebra Road
San Antonio, TX 78284

Dr. R. C. DeHart
Southwest Research Institute
8500 Culebra Road
San Antonio, TX 78284

Dr. M. L. Baron
Weidlinger Associates
110 East 59th Street
New York, NY 10022

Dr. T. L. Geers
Lockheed Missiles and Space Company
3251 Hanover Street
Palo Alto, CA 94304

Dr. J. L. Tocher
Boeing Computer Services
P.O. Box 24346
Seattle, WA 98124

Mr. William Caywood
Applied Physics Laboratory
Code BBE
8621 Georgia Avenue
Silver Spring, MD 20034

Dr. Robert E. Nickell
Pacifica Technology
P.O. Box 148
Del Mar, CA 92014

Dr. M. F. Kanninen
Battelle Columbus Laboratories
505 King Avenue
Columbus, OH 43201

Dr. G. T. Hahn
Battelle Columbus Laboratories
505 King Avenue
Columbus, OH 43201

Dr. A. A. Hochrein
Daedalean Associates, Inc.
Springlake Research Center
15110 Frederick Road
Woodbine, MD 21797

Dr. E. J. Tolle
IMASS, Inc.
Box 134
Accord, (Hingham) MA 02018

Dr. T. McDonough
Aeronautical Research Associates
of Princeton, Inc.
50 Washington Road
Princeton, NJ 08540

Mr. Richard Y. Dow
National Academy of Sciences
2101 Constitution Avenue
Washington, DC 20418

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13. ABSTRACT

A sinusoidal wave of infinitesimal amplitude is propagated in an isotropic compressible elastic material subjected to a static pure homogeneous deformation. Necessary and sufficient conditions are obtained for the speed of propagation to be real in the case when the direction of propagation is parallel to a principal plane of the static deformation.

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